Range of High-Energy Interactions*

FREDA SALZMAN
University of Colorado, Boulder, Colorado
(Received 15 February 1963)

The range of high-energy nucleon-nucleon interactions is examined in a simple single inelastic channel model which consists of the production of two spinless isobars. An almost transparent, purely absorbing optical approximation is made, in which case both the inelastic and elastic angular distributions are sensitive to the variation of the absorption coefficient η_l with angular momentum l. Unlike the case of the strong absorber, the inelastic and elastic interactions are described by different effective ranges. Two examples are given, one in which the inelastic channel has an angular distribution characteristic of a one-pion exchange process and the second in which it is characteristic of a "vacuum Regge pole" exchange.

I. INTRODUCTION

ON the basis of the total and elastic-differential nucleon-nucleon cross-section data from the large accelerators in the range 3–30 BeV/c incident laboratory nucleon momentum,[1] it seems reasonable to assume that the slowly varying total cross sections are approaching constant values and that the total elastic cross section is mainly due to diffraction effects. The observed variation with energy of the differential and total elastic cross section suggests that, as the energy increases, both the range and transparency of the nucleon-nucleon interaction are slowly increasing.[1,2] The rates of increase are such that the total cross section remains constant. There is considerable speculation[3] that the same behavior continues to very high energies, in which case the total elastic scattering becomes a small part of the total cross section and the nucleon-nucleon interaction becomes purely absorbing, almost transparent, and of very long range.

The basic inelastic nucleon-nucleon interactions are still not well understood. There is evidence from ~ 1 BeV/c laboratory momentum to the highest cosmic-ray energies that many events occur at large impact parameters, and correspondingly small momentum transfers. At the lower energies, it appears that the one-pion exchange interaction can reasonably explain the small momentum transfer or "peripheral" part of the interaction.[4] At the higher machine energies, there has been some suggestion that the one-pion exchange model may have to be modified and that other mechanisms are important in the small momentum transfer region.[5]

Since the elastic-differential cross sections are well known at the machine energies and since their behavior at higher energies has been conjectured, it would be helpful to have a better understanding of the type of diffraction pattern that even some of the simplest inelastic processes would give if they were dominant. An elementary "peripheral" model for the inelastic production is examined here and the elastic diffraction amplitude is obtained by the use of the unitarity relation in the almost transparent, purely absorbing optical approximation.

We assume that there is only one inelastic channel, which consists of the "peripheral" production of two "isobars" with known amplitudes. One of the "isobars" may be an unexcited nucleon. The spins of the isobars and incident nucleons are assumed to be negligible compared to the large angular momentum arising from the large relative velocity and the large impact parameter of the particles in the barycentric system.

In an optical model calculation it is usual (although not necessary) to assume that the medium is purely and uniformly absorbing up to some radius R, that is, the absorption coefficient η_l is real and constant for $l \leq L_{\max}$. This approximation is very good for the case of a black or almost black absorber for which $\eta_l \ll 1$. The inelastic and elastic cross sections are then almost equal and insensitive to the variation of η_l with l for $l \leq L_{\max}$. In the case of the single two-body inelastic channel considered here, the uniform or black absorbing interaction would give rise to the same angular distribution for the final-state particles in the elastic and inelastic channels.

What we are specifically interested in investigating here is an almost transparent, purely absorbing interaction with a long tail, which is the case in certain peripheral models. This approximation is most likely of interest in absorbing media for which the total elastic cross section is of the order of, or less than, one-tenth of the total cross section, $\sigma^{el} \lesssim \frac{1}{10}\sigma^{tot}$. Both the inelastic cross section and the elastic cross section are sensitive to the variation of the absorption coefficient η_l with l. For the simple two-body inelastic case, if η_l goes smoothly to 1 (no absorption) then the angular distri-

* Supported in part by a grant from the National Science Foundation.
[1] A. N. Diddens, H. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters **9**, 32, 108, and 111 (1962); and W. F. Baker, E. W. Jenkins, A. L. Read, G. Cocconi, V. T. Cocconi, and J. Orear, Phys. Rev. **9**, 221 (1962).
[2] B. M. Udgaonkar and M. Gell-Mann, Phys. Rev. Letters **8**, 346 (1962).
[3] V. N. Gribov, Zh. Eksperim. i Teor. Fiz. **41**, 667 (1961) [translation: Soviet Phys.—JETP **14**, 478 (1962)]; C. Lovelace, Nuovo Cimento **25**, 730 (1962); G. F. Chew and S. Frautschi, Phys. Rev. Letters **7**, 394 (1961) and **8**, 41 (1962); and R. Blankenbecler and M. L. Goldberger, Phys. Rev. **126**, 766 (1962).
[4] For a recent fit in p-p interactions and other references see G. B. Chadwick, G. B. Collins, P. J. Duke, T. Fujii, N. C. Hien, M. A. R. Kemp, and F. Turkot, Phys. Rev. **128**, 1823 (1962).
[5] S. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. **126**, 2204 (1962); and A. P. Contegouris, S. C. Frautschi, and H. Wong, Phys. Rev. **129**, 974 (1963).

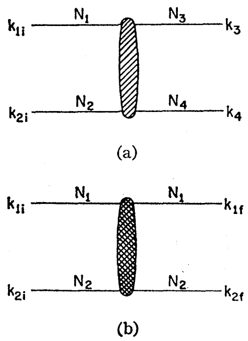

FIG. 1. Diagrammatic representations of reactions I and II. (a) Reaction I, in which particles N_1 and N_2 are incident with four-momenta k_{1i} and k_{2i} and particles N_3 and N_4 are produced with four-momenta k_3 and k_4. (b) Reaction II in which the two incident particles N_1 and N_2 scatter elastically and emerge with four-momenta k_{1f} and k_{2f}.

bution of the inelastic channel is narrower than that of the elastic channel. Thus, the effective range of the inelastic interaction is greater than that for the elastic one.

In Sec. II, we give the well-known equations obtained for this process from the unitarity condition and we amplify the above remarks.

Two explicit final-state angular distributions for the inelastic amplitudes are considered in Sec. III, one corresponding to a "one-pion exchange" interaction and the other to a "vacuum Regge pole" exchange. The elastic amplitudes are obtained for each case and the ranges of the two interactions are compared.

II. THE MODEL AND THE UNITARITY CONDITION

We assume (for generality) that two particles N_1 and N_2 are incident with four-momenta k_{1i} and k_{2i} and that their total interaction proceeds through two channels I and II,

$$N_1+N_2\rightarrow N_3+N_4 \qquad \text{I}$$

$$N_1+N_2\rightarrow N_1+N_2. \qquad \text{II}$$

Channel I represents a single inelastic interaction in which particles N_3 and N_4 are produced with four-momenta k_3 and k_4, as shown in Fig. 1(a). Channel II represents the associated elastic diffraction scattering required by unitarity, in which the two incident particles N_1 and N_2 come off with four-momenta k_{1f} and k_{2f}, as shown in Fig. 1(b). The masses of the particles are given by $k_i^2=m_i^2$, and units of $\hbar=c=1$ are used throughout.

The production and scattering angles in the barycentric system for channels I and II are θ_{fi}' and θ_{fi}, respectively, and are defined by

$$\mathbf{k}_3\cdot\mathbf{k}_{1i}=k'k\cos\theta_{fi}', \tag{2.1}$$

$$\mathbf{k}_{1f}\cdot\mathbf{k}_{1i}=k^2\cos\theta_{fi}, \tag{2.2}$$

where

$$|\mathbf{k}_{1i}|=|\mathbf{k}_{2i}|=|\mathbf{k}_{1f}|=|\mathbf{k}_{2f}|=k \quad\text{and}\quad |\mathbf{k}_3|=|\mathbf{k}_4|=k'.$$

The amplitudes for channels I and II are $f^{\rm I}(\theta_{fi}')$ and $f^{\rm II}(\theta_{fi})$, respectively, and are normalized so that the differential cross sections are given by

$$\frac{d\sigma^{\rm I}}{d\Omega_f'}=\frac{k'}{k}|f^{\rm I}(\theta_{fi}')|^2, \tag{2.3}$$

$$\frac{d\sigma^{\rm II}}{d\Omega_f}=|f^{\rm II}(\theta_{fi})|^2. \tag{2.4}$$

The particles are assumed to be spinless and the amplitude $f^{\rm I}(\theta_{fi}')$ is assumed to be known. The unitarity condition in terms of these amplitudes reduces to

$$\frac{1}{2i}[f^{\rm II}(\theta_{fi})-f^{\rm II*}(\theta_{fi})]=\frac{k}{4\pi}\int d\Omega_j f^{\rm II*}(\theta_{fj})f^{\rm II}(\theta_{ji}) +\frac{k'}{4\pi}\int d\Omega_n' f^{\rm I*}(\theta_{fn}')f^{\rm I}(\theta_{ni}'). \tag{2.5}$$

The phase of $f^{\rm II}$ has been defined in the usual manner so that its expansion into partial waves is given by

$$f^{\rm II}(\theta_{fi})=\frac{i}{2k}\sum_l(2l+1)f_l^{\rm II}P_l(\cos\theta_{fi}), \tag{2.6}$$

where

$$f_l^{\rm II}=1-\eta_l,$$

$\eta_l=e^{2i\delta_l}$, and δ_l is the phase shift.

Substitution of Eq. (2.6) into Eq. (2.5) gives

$$\frac{\pi}{k^2}\sum_l(2l+1)(1-|\eta_l|^2)P_l(\cos\theta_{fi}) =\frac{k'}{k}\int d\Omega_n' f^{\rm II*}(\theta_{fn}')f^{\rm II}(\theta_{ni}'), \tag{2.7}$$

where use has been made of the relation

$$\int d\Omega_j P_{l'}(\cos\theta_{fj})P_l(\cos\theta_{ji})=\delta_{ll'}\frac{4\pi}{2l+1}P_l(\cos\theta_{fi}). \tag{2.8}$$

Because $f^{\rm II}(\theta_{ni}')$ is a two-particle state amplitude it can be expanded simply into partial waves,

$$f^{\rm I}(\theta_{ni}')=\frac{1}{2(kk')^{1/2}}\sum_l(2l+1)f_l^{\rm I}P_l(\cos\theta_{ni}'), \tag{2.9}$$

where

$$f_l^{\rm I}=(kk')^{1/2}\int_{-1}^{1}f^{\rm II}(\theta_{ni}')P_l(\cos\theta_{ni}')d(\cos\theta_{ni}'), \tag{2.10}$$

and the phase of $f^{\rm I}(\theta_{ni}')$, which is not important, is taken equal to one.

Substitution of Eq. (2.9) into Eq. (2.7) and use of Eq. (2.8) gives

$$\textstyle\sum_l(2l+1)(1-|\eta_l|^2)P_l(\cos\theta_{fi}) =\sum_l(2l+1)|f_l^{\rm I}|^2P_l(\cos\theta_{fi}), \tag{2.11}$$

which implies

$$1-|\eta_l|^2=|f_l^{\rm I}|^2. \tag{2.12}$$

The last equation gives the well-known unitarity limits

$$0 \leq |\eta_l|^2 \leq 1, \quad 0 \leq |f_l^{\mathrm{I}}|^2 \leq 1. \tag{2.13}$$

If we now assume that η_l is pure real, we have

$$f_l^{\mathrm{I}} = [1-\eta_l^2]^{1/2}, \quad f_l^{\mathrm{II}} = 1-\eta_l. \tag{2.14}$$

From these equations the statements made in the Introduction follow very simply. For a black or almost black absorber $\eta_l \ll 1$ for $l \leq L_{\max}$ and $f_l^{\mathrm{I}} \approx f_l^{\mathrm{II}}$. If f_l^{I} and f_l^{II} drop rapidly to zero for $l > L_{\max}$, we can ignore the contribution of these partial waves, at least for the small angle scattering where all the waves are in phase, and we have

$$f^{\mathrm{I}}(\theta_{fi}') = \frac{1}{2(kk')^{\frac{1}{2}}} \sum_{l=0}^{L_{\max}} (2l+1) P_l(\cos\theta_{fi}'), \quad f^{\mathrm{II}}(\theta_{fi}) = \frac{i}{2k} \sum_{l=0}^{L_{\max}} (2l+1) P_l(\cos\theta_{fi}). \tag{2.15}$$

Thus, the angular distribution of the elastic diffraction pattern is the same as that of the two-body inelastic channel. The uniform sharp-edge absorber gives the well-known diffraction pattern with varying intensity beyond the first diffraction minimum. This effect is very model-dependent and not present if there is some "real scattering" or if the edge of the absorber goes to zero sufficiently slowly. The small angle scattering remain essentially the same.[6]

If $\eta_l \approx 1$, we take $\eta_l = 1-\epsilon_l$ where $\epsilon_l \ll 1$ and Eq. (2.14) gives

$$f_l^{\mathrm{I}} = [2\epsilon_l]^{1/2}, \quad f_l^{\mathrm{II}} = \epsilon_l. \tag{2.16}$$

The approximation used in obtaining the expression for f_l^{I} in Eq. (2.16) is what we mean by the almost transparent approximation, and is used in the next section to obtain the amplitude f_l^{II} from f_l^{I}. It is reasonable for values of $\epsilon_l \lesssim 0.2$ or, in terms of the cross sections (which are proportional to the squares of the amplitudes), for $\sigma_l^{\mathrm{II}} \lesssim \frac{1}{10}\sigma_l^{\mathrm{I}}$.

The amplitudes f_l^{I} and f_l^{II} are dependent upon the variation of ϵ_l. If ϵ_l goes smoothly to zero, then for sufficiently large l, $f_l^{\mathrm{I}} \gg f_l^{\mathrm{II}}$. The angular distribution in channel I is narrower than that of the diffraction scattering in II, and the corresponding effective ranges R^{I} and R^{II} are different, $R^{\mathrm{I}} > R^{\mathrm{II}}$.

Another relation which is useful is the optical theorem for the forward elastic amplitude $f^{\mathrm{II}}(0)$. From Eqs. (2.3), (2.4), and (2.5) we obtain

$$\frac{1}{2i}[f^{\mathrm{II}}(0) - f^{\mathrm{II}*}(0)] = \frac{k}{4\pi}(\sigma^{\mathrm{I}} + \sigma^{\mathrm{II}}). \tag{2.17}$$

III. TWO EXAMPLES

A. One-Pion Exchange

For the exchange of a spinless particle of mass m, the angular distribution in channel I is given by

$$f^{\mathrm{I}}(\theta_{3i}) = \frac{(m_1 m_2 m_3 m_4)^{1/2}}{2\pi s^{\frac{1}{2}}} \Lambda_1 \Lambda_2 \frac{1}{m^2 - t}, \tag{3.1}$$

where $s^{1/2}$ is the total center of mass energy, $s^2 = (k_1+k_2)^2$, Λ_1 and Λ_2 are vertex functions, and t is given by

$$t = (k_3 - k_1)^2 = -[(\Delta^2)_{\min} + 2kk'(1-\cos\theta_{fi}')], \tag{3.2}$$

where $(\Delta^2)_{\min}$ is a function of m_1^2, m_2^2, m_3^2, m_4^2, and s. For each of $(\Delta^2)_{\min}$, m_1^2, m_2^2, m_3^2, $m_4^2 \ll s$, and m_3^2 and m_4^2 sufficiently greater than m_1^2 and m_2^2, respectively, $(\Delta^2)_{\min}$ is well approximated by

$$(\Delta^2)_{\min} = (m_3^2 - m_1^2)(m_4^2 - m_2^2)/s.$$

It is assumed that Λ_1 and Λ_2 have only weak dependence on θ_{fi}'. In terms of the angle θ_{fi}', $f^{\mathrm{I}}(\theta_{fi}')$ is

$$f^{\mathrm{I}}(\theta_{fi}') = f^{\mathrm{I}}(0) \frac{\frac{1}{2}\eta^2}{1+\frac{1}{2}\eta^2 - \cos\theta_{fi}'}, \tag{3.3}$$

where $f^{\mathrm{I}}(0) = (m_1 m_2 m_3 m_4)^{1/2} \Lambda_1 \Lambda_2 / 2\pi s^{1/2} (kk')\eta^2$, and $\eta^2 = m^2 + (\Delta^2)_{\min}/kk'$. From Eq. (2.10) we then obtain for f_l^{I}

$$f_l^{\mathrm{I}} = (kk')^{1/2} \eta^2 f^{\mathrm{I}}(0) Q_l(1+\tfrac{1}{2}\eta^2),$$

where $Q_l(a)$ is the Legendre function of the second kind defined by

$$Q_l(a) = \frac{1}{2}\int_{-1}^{+1} \frac{P_l(x)}{a-x} dx, \quad \text{for} \quad a > 1.$$

From the approximation given in Eq. (2.16) we have

$$f_l^{\mathrm{II}} = \tfrac{1}{2}(f_l^{\mathrm{I}})^2 = \tfrac{1}{2}kk'\eta^4 [f^{\mathrm{I}}(0)]^2 Q_l^2(1+\tfrac{1}{2}\eta^2). \tag{3.4}$$

For $\eta \ll 1$

$$Q_l(1+\tfrac{1}{2}\eta^2) = K_0(l\eta), \tag{3.5}$$

and for $l\eta \gg 1$

$$K_0(l\eta) = \left(\frac{\pi}{2}\right)^{1/2} \frac{e^{-l\eta}}{(l\eta)^{1/2}}.$$

[6] An illustration of these remarks is contained in the paper of T. Fujii, G. B. Chadwick, G. B. Collins, P. J. Duke, N. C. Hien, M. A. R. Kemp, and F. Turkot, Phys. Rev. **128**, 1836 (1962), in which they analyze p-p elastic scattering data in the 1–3 BeV range. They find, in particular, at 1.35-BeV incident nucleon laboratory kinetic energy, that $\sigma^{\mathrm{el}} \approx \sigma^{\mathrm{in}} \approx \frac{1}{2}\sigma^{\mathrm{tot}}$, where σ^{el} is the total elastic cross section, σ^{in} the total inelastic, and σ^{tot} the combined total cross section. On the assumption of a purely absorbing medium, this implies almost complete absorption. They also find at small angles essentially the same angular distribution as given by Eq. (2.15) for the elastic scattering and a dominant "two-body" peripheral inelastic channel. It is necessary to include a small amount of "real scattering" to make the large angle diffraction pattern agree with the smooth tail of the experimental data.

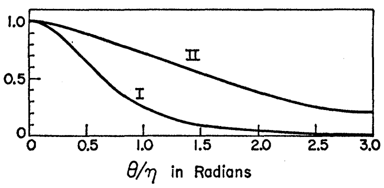

FIG. 2. Barycentric differential cross sections for example A. Curve I is the barycentric differential cross section for reaction I given by Eq. (3.3) and II is that for reaction II given by Eq. (3.13). The cross sections are normalized to one in the forward direction.

The amplitudes f_l^{I} and f_l^{II} become

$$f_l^{\mathrm{I}} \propto e^{-l\eta}/(l\eta)^{1/2},$$

and

$$f_l^{\mathrm{II}} \propto e^{-2l\eta}/l\eta \quad \text{for} \quad \eta \ll 1 \quad \text{and} \quad l\eta \gg 1.$$

The exponential factor contains the main l dependence. The important l values for channels I and II are given by $l \lesssim L^{\mathrm{I}}, L^{\mathrm{II}}$, respectively, where

$$L^{\mathrm{I}} = \frac{1}{\eta} = \left[\frac{kk'}{m^2 + (\Delta^2)_{\min}}\right]^{1/2},$$

and

$$L^{\mathrm{II}} = \frac{1}{2\eta} = \frac{1}{2}\left[\frac{kk'}{m^2 + (\Delta^2)_{\min}}\right]^{1/2}. \tag{3.6}$$

The actual definition of range in terms of impact parameter appears ambiguous for channel I because there are two momenta present, k and k'. For the elastic case, channel II, it is usual to let $R^{\mathrm{II}} = L^{\mathrm{II}}/k$ and if we define R^{I} likewise, $R^{\mathrm{I}} = L^{\mathrm{I}}/k$, we have

$$R^{\mathrm{II}} = \tfrac{1}{2}R^{\mathrm{I}} = \frac{1}{2}\left[\frac{k'/k}{m^2 + (\Delta^2)_{\min}}\right]^{1/2}. \tag{3.7}$$

We see that $(\Delta^2)_{\min}$ is important in determining the effective range. It reduces the range and acts to increase the effective mass of the exchanged particle. For $(\Delta^2)_{\min} \ll m^2$ and $k' \approx k$, we obtain the longest range possible for the process in this approximation

$$R^{\mathrm{II}} = \tfrac{1}{2}R^{\mathrm{I}} = 1/2m.$$

If the absorption were strong in channel I, then one expects the maximum ranges to be

$$R^{\mathrm{II}} = R^{\mathrm{I}} = 1/m.$$

The elastic scattering amplitude obtained directly from Eqs. (2.6) and (3.4) is

$$f^{\mathrm{II}}(\theta_{fi}) = \frac{i}{4}k'\eta^4[f^{\mathrm{I}}(0)]^2 \sum_l (2l+1)Q_l^2\left(1+\frac{\eta^2}{2}\right)P_l(\cos\theta_{fi}).$$

For $\theta_{fi} \ll 1$,

$$P_l(\cos\theta_{fi}) = J_0(l\theta_{fi}), \tag{3.8}$$

and with the use of Eq. (3.5) for small η, $\eta \ll 1$, we obtain

$$f^{\mathrm{II}}(\theta_{fi}) = \frac{i}{4}k'\eta^4[f^{\mathrm{I}}(0)]^2\{\sum_l (2l+1)K_0^2(l\eta)J_0(l\theta_{fi})\}. \tag{3.9}$$

For small η many l values contribute and the sum in the bracket can be replaced by an integral over l given by[7]

$$2\int_0^\infty lK_0^2(l\eta)J_0(l\theta_{fi})dl = 2\frac{1}{(4\eta^2+\theta_{fi}^2)^{1/2}}\frac{1}{\theta_{fi}} \times \ln\left[\frac{(4\eta^2+\theta_{fi}^2)^{1/2}+\theta_{fi}}{(4\eta^2+\theta_{fi}^2)^{1/2}-\theta_{fi}}\right].$$

Equation (3.9) can be written as

$$f^{\mathrm{II}}(\theta_{fi}) = f^{\mathrm{II}}(0)\frac{2\eta^2}{(4\eta^2+\theta_{fi}^2)^{1/2}}\frac{1}{\theta_{fi}} \times \ln\left[\frac{(4\eta^2+\theta_{fi}^2)^{1/2}+\theta_{fi}}{(4\eta^2+\theta_{fi}^2)^{1/2}-\theta_{fi}}\right], \tag{3.10}$$

where

$$f^{\mathrm{II}}(0) = \frac{i}{4}k'\eta^2[f^{\mathrm{I}}(0)]^2. \tag{3.11}$$

Substitution of Eq. (3.10) into Eq. (2.4) and of Eq. (3.3) into Eq. (2.3) then gives for the differential cross sections

$$\frac{d\sigma^{\mathrm{I}}}{d\Omega_f'} = \frac{k'}{k}[f^{\mathrm{I}}(0)]^2\left[\frac{\frac{1}{2}\eta^2}{1+\frac{1}{2}\eta^2-\cos\theta_{fi}'}\right]^2 \tag{3.12}$$

and

$$\frac{d\sigma^{\mathrm{II}}}{d\Omega_f} = [f^{\mathrm{II}}(0)]^2\left(\frac{4\eta^4}{4\eta^2+\theta_{fi}^2}\right)\frac{1}{\theta_{fi}^2} \times \left[\ln\frac{(4\eta^2+\theta_{fi}^2)^{1/2}+\theta_{fi}}{(4\eta^2+\theta_{fi}^2)^{1/2}-\theta_{fi}}\right]^2. \tag{3.13}$$

For $\theta^2 \ll 4\eta^2 \ll 1$, Eqs. (3.12) and (3.13) reduce to

$$\frac{d\sigma^{\mathrm{I}}}{d\Omega_f'} = \frac{k'}{k}[f^{\mathrm{I}}(0)]^2\frac{\eta^4}{(\theta_{fi}'^2+\eta^2)^2},$$

and

$$\frac{d\sigma^{\mathrm{II}}}{d\Omega_f} = [f^{\mathrm{II}}(0)]^2\frac{4\eta^4}{(\theta_{fi}^2+(2\eta)^2)^2}, \tag{3.14}$$

and the difference in the angular distributions is apparent. The differential cross sections given by Eqs. (3.12) and (3.13) are shown plotted in Fig. 2 as curves I and II, respectively. The elastic diffraction cross section falls off very slowly in the range $\theta_{fi} \gtrsim \eta$ due to the logarithmic dependence.

[7] *Tables of Integral Transforms, Bateman Manuscript Project*, edited by A. Erdelyi (McGraw-Hill Book Company, Inc., New York, 1954), Vol. 2, p. 16.

We now determine the extent to which the optical theorem as given by Eq. (2.17) is satisfied. From Eqs. (3.10) and (3.11) we have

$$\frac{1}{2i}[f^{\mathrm{II}}(0)-f^{\mathrm{II}*}(0)]=\tfrac{1}{4}k'\eta^2[f^{\mathrm{I}}(0)]^2.$$

Using Eq. (3.12), we find

$$\frac{k}{4\pi}\sigma^{\mathrm{I}}=\tfrac{1}{4}k'\eta^2[f^{\mathrm{I}}(0)]^2,$$

where we have assumed that for $\eta^2\ll\theta^2\ll1$ the cross section goes to zero. The optical theorem is saturated by σ^{I} with the approximations made, so that it is satisfied in the limit $\sigma^{\mathrm{II}}\ll\sigma^{\mathrm{I}}$.

B. Vacuum Regge-Pole Exchange

The amplitude of reaction I, if due to the exchange of a "vacuum Regge pole," is given by

$$f^{\mathrm{I}}(\theta_{fi}')=\frac{(m_1m_2m_3m_4)^{1/2}}{2\pi s^{1/2}}g_1g_2\left(\frac{s}{2m_1m_2}\right)^{\alpha(t)},\quad(3.15)$$

where g_1 and g_2 are vertex functions which are assumed not to have important t dependence. In the linear approximation[2]

$$\alpha(t)=\alpha(0)+\alpha' t.\quad(3.16)$$

Using this expression and that for t given in Eq. (3.2) we obtain

$$f^{\mathrm{I}}(\theta_{fi}')=f^{\mathrm{I}}(0)e^{-\beta(1-\cos\theta_{fi}')},\quad(3.17)$$

where

$$f^{\mathrm{I}}(0)=\frac{(m_1m_2m_3m_4)^{1/2}}{2\pi s^{1/2}}g_1g_2e^{\gamma},$$

$$\gamma=\left(\ln\frac{s}{2m_1m_2}\right)[\alpha(0)-\alpha'(\Delta^2)_{\min}],$$

and

$$\beta=2kk'\alpha'\left(\ln\frac{s}{2m_1m_2}\right).$$

Making the small angle approximation $P_l(\cos\theta)=J_0(l\theta)$ in Eq. (2.9), we have

$$f_l^{\mathrm{I}}=(kk')^{1/2}f^{\mathrm{I}}(0)\int_{-1}^{1}e^{-\beta(1-\cos\theta_{fi}')}J_0(l\theta_{fi}')d(\cos\theta_{fi}'),$$

$$=(kk')^{1/2}\frac{f^{\mathrm{I}}(0)}{\beta}\int_0^{2\beta^{1/2}}e^{-x^2/2}J_0(lx/\beta^{1/2})xdx,$$

where $x^2/2=\beta(1-\cos\theta_{fi}')$ and $J_0(l\theta_{fi}')=J_0(lx/\beta^{1/2})$. The major contribution to the integral comes from values of $\theta_{fi}'=x/\beta^{1/2}\ll1$. Thus, for large enough β the upper limit can be taken as ∞. We then have[8,2]

$$f_l^{\mathrm{I}}=(kk')^{1/2}\frac{f^{\mathrm{I}}(0)}{\beta}e^{-l^2/2\beta}.\quad(3.18)$$

From Eq. (2.14) we obtain

$$f_l^{\mathrm{II}}=\tfrac{1}{2}(f_l^{\mathrm{I}})^2=kk'\frac{[f^{\mathrm{I}}(0)]^2}{2\beta^2}e^{-l^2/\beta}.\quad(3.19)$$

We define the maximum value for the two channels as previously by the exponential fall off,

$$L^{\mathrm{II}}=L^{\mathrm{I}}/\sqrt{2}=\beta^{1/2}=\left[2kk'\alpha'\ln\frac{s}{2m_1m_2}\right]^{1/2},\quad(3.20)$$

and correspondingly (the ranges)

$$R^{\mathrm{II}}=R^{\mathrm{I}}/\sqrt{2}=\left[\frac{2k'}{k}\alpha'\ln\frac{s}{2m_1m_2}\right]^{1/2}.$$

R^{I} and R^{II} are closer in value for this example than the previous one because the amplitudes for the Regge-pole exchange interaction decrease more rapidly with l than those for the one pion exchange interaction.

Substituting Eq. (3.19) into Eq. (2.6), and noting that f_l^{II} and f_l^{I} have the same functional dependence on l, we obtain

$$f^{\mathrm{II}}(\theta_{fi})=f^{\mathrm{II}}(0)\exp[-(\beta/2)(1-\cos\theta_{fi})],\quad(3.21)$$

where

$$f^{\mathrm{II}}(0)=2k'\frac{[f^{\mathrm{I}}(0)]^2}{4\beta}.$$

The momentum transfer variable in channel II is given by

$$t=-2k^2(1-\cos\theta_{fi}),$$

and $f^{\mathrm{II}}(\theta_{fi})$ can be written as

$$f^{\mathrm{II}}(\theta_{fi})=f^{\mathrm{II}}(0)\exp\{(k'/2k)\alpha'[\ln(s/2m_1m_2)]t\}.\quad(3.22)$$

If one starts with an angular distribution characteristic of the exchange of the vacuum Regge trajectory with a slope α' in channel I, then in this approximation the elastic diffraction amplitude can be described as the exchange of a trajectory with slope $(k'/2k)\alpha'$. It is clear that even for $k'\approx k$, with the approximations used here, channels I and II cannot be described in terms of the exchange of Regge trajectories of the same slope, not to mention the possibility of the same trajectory. Comparison of Eqs. (3.21) and (3.17) show that for small angles, the amplitudes are characterized by widths $\theta_{fi}'^2\lesssim2/\beta$ and $\theta_{fi}^2\lesssim4/\beta$.

Finally, we again check the consistency of our results with the optical theorem as given by Eq. (2.17). We

[8] *Tables of Integral Transforms, Bateman Manuscript Project*, edited by A. Erdelyi (McGraw-Hill Book Company, Inc., New York, 1954), Vol. 2, p. 9.

have from Eqs. (2.3) and (3.17)

$$\sigma^{\mathrm{I}}=[f^{\mathrm{I}}(0)]^2\frac{\pi}{\beta}\quad\text{for large }\beta.$$

Thus,

$$\frac{k}{4\pi}\sigma^{\mathrm{I}}=k'[f^{\mathrm{I}}(0)]^2\frac{1}{4\beta},$$

and from Eq. (3.22) we have

$$\frac{f^{\mathrm{II}}(0)-f^{\mathrm{II}*}(0)}{2i}=k'[f^{\mathrm{I}}(0)]^2\frac{1}{4\beta}.$$

The optical theorem is satisfied in the limit $\sigma^{\mathrm{II}}\ll\sigma^{\mathrm{I}}$.

IV. DISCUSSION

The production of two isobars is not expected to be an important reaction in very high energy nucleon-nucleon collisions. It is given here only as an example in which the effective range in the inelastic channel is larger than that in the elastic channel for the case of a "peripheral" interaction in the almost transparent, purely absorbing approximation. For more complicated final states, the effective range is related to the total angular momentum, which is compounded from that of each of the "particles" and the problem requires a more general treatment.[9]

However, in chain- or "linked"-peripheral processes, which are of interest at very high energy,[10] a considerable simplification can be made.[11] If the internal "isobars" or "fireballs" that are produced are assumed to have suitably defined small energy, then, in first approximation, they serve to increase the range of the interaction of the end isobars but carry off only a small part of the total angular momentum themselves. The amplitude can be assumed to depend only on the angular variables of the two end isobars through a modified propagator or exchange mechanism. The process then simulates the "two isobar" case considered here, and the angular dependence of the approximate amplitude is the same as that given by Eq. (3.17).[11] Thus, as suggested by the "Regge-pole" exchange interaction example, the elastic diffraction scattering has the characteristic exponentially decreasing dependence on momentum transfer, if there is present a dominant long-range interaction of the same type (but of greater range) in an inelastic channel.

[9] *Note added in proof.* In a recent paper, L. Van Hove [CERN-5445/TH320 (unpublished)] discusses phenomenologically inelastic collisions at high energies. He considers simple forms of the wave function of the inelastic final state and determines under which conditions they are compatible through unitarity with the known or conjectured properties of diffraction scattering.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor Miesowicz and his group for discussions which served to initiate this investigation and for the cordial hospitality extended to me at the Akademia Gorniczo-Hutnicza. During this time, I was a National Science Foundation Postdoctoral Fellow and I wish to thank Professor Cini for the warm hospitality extended to me at the University of Rome and Professor L. Van Hove for that extended to me while at CERN. I also wish to thank Professor T. Kinoshita for useful discussions. I am indebted to Professor G. Salzman for discussions, his encouragement, and for a critical reading of this manuscript.

[10] S. C. Frautschi, Nuovo Cimento **28**, 409 (1963).

[11] F. Salzman (to be published).